

ON QUASIDEGENERACY OF MAJORANA NEUTRINOS AND THE OBSERVED PATTERN OF LEPTONIC MIXING

M. N. Rebelo^a

*Centro de Física Teórica de Partículas - CFTP and Departamento de Física,
Instituto Superior Técnico - IST, Universidade de Lisboa, Av. Rovisco Pais,
P-1049-001 Lisboa, Portugal*

Abstract. We relate the observed pattern of leptonic mixing to the quasidegeneracy of three Majorana neutrinos. We show how lifting the degeneracy may lead to the measured value of $|U_{13}|$ and to sizeable CP violation of Dirac-type. We show some of the correlations obtained among physical observables, starting from some of the most interesting schemes proposed in the literature.

1 Introduction

This talk is based on work done in collaboration with G. C. Branco, J. I. Silva-Marcos and Daniel Wegman [1].

It was already shown, quite some time ago that the limit of exact degeneracy of Majorana neutrinos is non trivial in the sense that it allows for leptonic mixing and CP violation [2]. This result relies on the assumption that neutrinos are Majorana particles. Only fermions that are neutral under all $U(1)$ type symmetries can have Majorana character [3], therefore all other fermions in the Standard Model (SM) are Dirac fermions. Despite great recent experimental progress in the field of Neutrino Physics, the question of whether neutrinos are Dirac or Majorana fermions still remains one of the major open questions in Particle Physics.

The 2015 Nobel prize was awarded to T. Kajita and B. McDonald “for the discovery of neutrino oscillations, which shows that neutrinos have mass”. In fact, it is by now established that at least two of the observed neutrinos have non-zero masses. The presently allowed ranges are listed in Table 1. It should be noted that the sign of $\Delta m_{31}^2 \equiv m_3^2 - m_1^2$, is not yet known and is at present under intense experimental investigation. Experimentally the mass of the lightest neutrino, either ν_1 or ν_3 is still consistent with zero. However, upper bounds for the neutrino masses allow for quasidegeneracy. It is therefore of great interest to consider the limit of exact degeneracy of neutrino masses and possible ways of lifting this degeneracy. Specially so because the limit of exact degeneracy favours two large mixing angles and leads to one zero mixing angle which upon the lifting of the degeneracy can give rise to the observed small angle. The leptonic mixing angles have been measured experimentally, and are also listed in Table 1. There are two angles that are large when compared to the Cabibbo angle and one small mixing angle, θ_{13} , which was consistent with zero until the recent measurements performed at reactor [4] and accelerator neutrino experiments [5]. Leptonic mixing is remarkably different from

^aE-mail: rebelor@tecnico.ulisboa.pt

quark mixing. It is not yet known whether or not there is CP violation in the leptonic sector either at low or at high energies [6]. In the absence of a flavour model no relation can be established between these two manifestations [7], [8]. The limit of exact degeneracy with Majorana neutrinos allows for one Majorana-type CP violating phase in the mixing matrix [2]. Lifting of the degeneracy together with the requirement that all three mixing angles should be different from zero allows for Dirac as well as Majorana-type CP violation [1]. In Ref. [1] we studied perturbations of exact degeneracy around some of the very well known mixing textures proposed in the literature, with $\theta_{13} = 0$, [9], [10], [11], [12], [13], [14], [15], [16] leading to quasidegenerate masses and mixing in agreement with all available experimental data. We also showed that in this process it is possible to generate Dirac-type CP violation large enough in strength to be detectable in the next round of neutrino experiments without introducing new sources of CP violation.

2 Present Experimental Knowledge of Neutrino Masses and Leptonic Mixing

Table 1 gives the 2014 update on global fits of neutrino oscillation parameters provided by Forero, Tortola and Valle [17]. The quantities Δm_{ij}^2 are defined by $(m_i^2 - m_j^2)$ and the angle θ_{ij} and the phase δ are those of the standard parametrisation [18]

Table 1: Neutrino oscillation parameter summary. For Δm_{31}^2 , $\sin^2 \theta_{23}$, $\sin^2 \theta_{13}$, and δ the upper (lower) row corresponds to normal (inverted) neutrino mass hierarchy. ^aThere is a local minimum in the first octant, $\sin^2 \theta_{23} = 0.467$ with $\Delta\chi^2 = 0.28$ with respect to the global minimum. From [17]

Parameter	Best fit	1σ range
Δm_{21}^2 [$10^{-5}eV^2$]	7.60	7.42 – 7.79
$ \Delta m_{31}^2 $ [$10^{-3}eV^2$](NH)	2.48	2.41 – 2.53
$ \Delta m_{31}^2 $ [$10^{-3}eV^2$](IH)	2.38	2.32 – 2.43
$\sin^2 \theta_{12}$	0.323	0.307 – 0.339
$\sin^2 \theta_{23}$ (NH)	0.567	0.439 ^a – 0.599
$\sin^2 \theta_{23}$ (IH)	0.573	0.530 – 0.598
$\sin^2 \theta_{13}$ (NH)	0.0234	0.0214 – 0.0254
$\sin^2 \theta_{13}$ (IH)	0.0240	0.0221 – 0.0259
δ (NH)	1.34 π	0.96 – 1.98 π
δ (IH)	1.48 π	1.16 – 1.82 π

Several textures for the leptonic mixing have been studied in the literature, often in the context of family symmetries [19], [20], [21]. In most of the proposed

schemes, the pattern of leptonic mixing is predicted but the spectrum of masses is not constrained by the symmetries. It is therefore consistent to consider these schemes, together with the hypothesis of quasidegeneracy of Majorana neutrinos.

3 The Limit of Exact Degeneracy with Majorana Neutrinos

We work in the framework of three left-handed neutrinos and write the effective Majorana mass term in the weak basis where the charged lepton mass matrix is diagonal, real and positive as:

$$\mathcal{L}_{\text{mass}} = - (\nu_{L\alpha})^T C^{-1} (M_o)_{\alpha\beta} \nu_{L\beta} + \text{h.c.} \quad (1)$$

where $\nu_{L\alpha}$ stand for the left-handed weak eigenstates and M_o is a 3×3 symmetric complex mass matrix. Obviously there is no loss of generality in choosing this weak-basis. In general M_o is diagonalized by a unitary matrix U_o through $U_o^T M_o U_o = \text{diag} (m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$. In the limit of exact degeneracy, M_o can be written:

$$M_o = \mu S_o \quad (2)$$

where μ is the common neutrino mass and $S_o = U_o^* U_o^\dagger$. Therefore, in the limit of exact degeneracy M_o is proportional to a symmetric unitary matrix. Leptonic mixing and even CP violation can occur in this limit provided that neutrinos are Majorana particles [2]. Leptonic mixing can be rotated away if and only if there is CP invariance and all neutrinos have the same CP parity [22]. It was also shown in Ref [2] that in the case of different CP parities the leptonic mixing matrix U_o can be parametrised by two angles and one phase, in the form:

$$U_o = O_{23}(\phi) O_{12}\left(\frac{\theta}{2}\right) \begin{pmatrix} 1 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & e^{-i\frac{\alpha}{2}} \end{pmatrix} \quad (3)$$

up to an orthogonal rotation of the three degenerate neutrinos and with each orthogonal matrix O_{ij} chosen to be symmetric. Obviously, if U_o diagonalises M_o so does $U_o O$ with O an arbitrary orthogonal rotation. An important feature is the fact that U_o always has one zero entry which in the above parametrisation appears in the (13) position. Although the location of the zero is not fixed the observed pattern of leptonic mixing suggests that the above choice is a good starting point for a perturbation.

4 Lifting the Degeneracy

In order to lift the degeneracy one may add a small perturbation to S_o :

$$M = \mu (S_o + \varepsilon^2 Q_o) \quad (4)$$

where the matrix Q_0 is fixed in such a way that the correct neutrino masses are obtained. We assume that after lifting the degeneracy the leptonic mixing matrix is given by:

$$U_{PMNS} = U_o \cdot O \quad (5)$$

where O is an orthogonal matrix, parametrized by small angles. The matrix O is denoted by:

$$O = O_{12}O_{13}O_{23} = \begin{pmatrix} c_{\phi_1} & s_{\phi_1} & 0 \\ -s_{\phi_1} & c_{\phi_1} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{\phi_3} & 0 & s_{\phi_3} \\ 0 & 1 & 0 \\ -s_{\phi_3} & 0 & c_{\phi_3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\phi_2} & s_{\phi_2} \\ 0 & -s_{\phi_2} & c_{\phi_2} \end{pmatrix} \quad (6)$$

Notice that U_{PMNS} still diagonalises S_o , thus establishing a strong connection between the degenerate and quasidegenerate case. Different cases were analysed by choosing U_0 to coincide with some of the most interesting cases considered in the literature with a zero in the (13) entry.

One of the examples considered in Ref. [1] consists of perturbing the tribimaximal mixing. In this case, we have: $U_o = U_{TBM} \cdot K$ with:

$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \quad \text{and} \quad K = \text{diag}(1, i, e^{-i\alpha/2}) \quad (7)$$

In the notation of Eq. (3), this ansatz corresponds to $\phi = 45^\circ$ and $\cos(\frac{\theta}{2}) = \frac{2}{\sqrt{6}}$ i.e., $\frac{\theta}{2} = 35.26^\circ$. We used data from the global fit of neutrino oscillations provided in 2012 by the authors of Ref. [17], requiring agreement within 1σ range. Although there are deviations in the more recent data from that of 2012 the deviations are slight and therefore the conclusions would not change significantly. In Ref. [1] we concluded that, in this example, complying with the experimental bounds allowed the leptonic strength of Dirac-type CP violation to range from 0 to about 4×10^{-2} , so that it could be within reach of future neutrino experiments.

Here we reproduce one of the figures obtained in the previous reference showing the correlation between I_{CP} and $|U_{13}|^2$. This scenario allows for a particularly simple solution since, one can reach agreement with the experimental data by choosing a matrix O with only one parameter different from zero, namely the angle ϕ_2 .

5 Final Remarks

The starting point of our analysis is the limit of exact degeneracy of neutrino masses. In this limit there is no Dirac-type CP violation, only Majorana-type CP violation may occur and one of the leptonic mixing angles is exactly zero. It

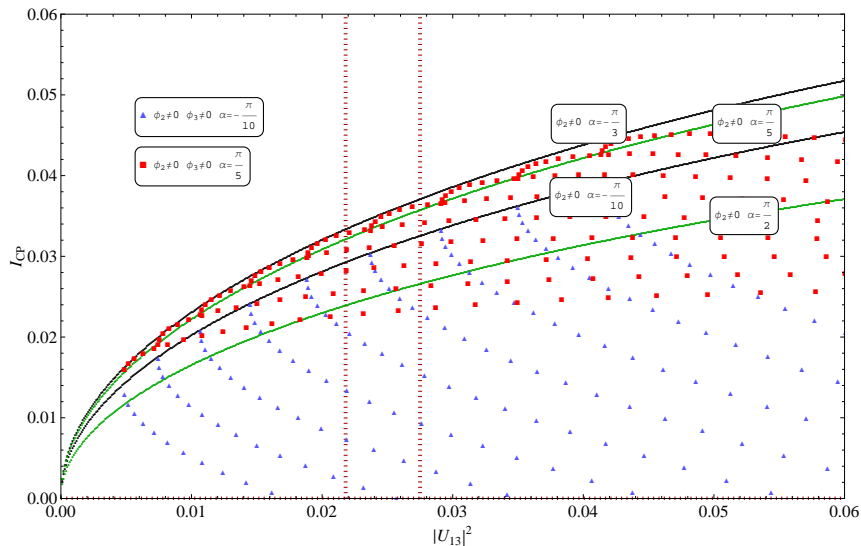


Figure 1: I_{CP} versus $|U_{13}|^2$ obtained by perturbing tribimaximal mixing with $\phi_3 = 0$. Each curve corresponds to a fixed α and to $\phi_1 = 0$, therefore ϕ_2 is the only variable. The points drifting away from each curve were obtained by varying also ϕ_3 .

has been shown in Ref. [23] that there is no loss of generality in parametrising a general unitarity matrix in the form given by Eq.(5), with U_0 of the generic form introduced before, however fixing the parameters of U_0 , as was done in the previous analysis, by making use of some of the most interesting patterns discussed in the literature with $\theta_{13} = 0$ restricts U_{PMNS} , and gives rise to predictive power while at the same time allowing to relate quasidegeneracy of neutrino masses with the observed pattern of leptonic mixing..

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